JACAMaC Individual Algebra Questions - 10 Questions, 45 minutes, No Calculator. The point values for each question are in parentheses.

1. (3) If half of a number is 5 more than a third of the same number, what is the number?

Answer: 30

Explanation: x/2 = 5 + x/3 => 3x = 30 + 2x => x = 30.

2. (3) Points ***P*** and ***R*** are located at (0,3) and (8,17) respectively. Point M is the midpoint of segment $\overline{PR}$. Segment $\overline{PR}$ is reflected over the *y*-axis. What is the sum of the coordinates of the image (reflected point) of point M?

Answer: 6

Explanation: First you can find the midpoint of PR -> ((0 + 8)/2 , (3 + 17)/2) -> (4,10). When reflected over the y-axis, the x value becomes negative, so the image is at point (-4,10), which has a sum of 6.

3. (3) What is the slope of a line perpendicular to the line $3x - 2y = -23$? (The slope can be expressed in the form $\frac{a}{b}$; express your answer as |ab|. Example: Express the answer of $-\frac{17}{14}$ as 1714.)

Answer: 23

Explanation: First you need to change the equation to slope-intercept form (y = mx + b) -> -2y = -3x - 23 . Now solve for y -> y = (3/2)x + 23/2 . The slope of this line is therefore 3/2. To find the slope of a perpendicular line, find the negative reciprocal of the slope (3/2 -> -⅔). Taking the absolute value of the concatenation of this slope gives an answer of 23.

4. (3) John is choosing between which rental car he should use to travel to New York. The pickup truck holds 25 gallons of gas but only gets 15 miles per gallon. The minivan holds 17 gallons of gas and gets 23 miles per gallon. The sedan holds 12 gallons of gas and gets 35 miles per gallon. The smart car holds only 8 gallons of gas but gets 50 miles per gallon. What is the farthest distance John can travel (in miles) on a full tank using one of the rental cars?

Answer: 420

Explanation: The farthest you can travel in a car is its gas mileage times the number of gallons it can hold. The sedan can travel the farthest, allowing you to go 35 x 12 = 420 miles.

5. (4) 10 years ago, John's grandfather was twice the age of John's father, who in turn was twice the age of John. John is 63 years younger than his grandfather. How old is John today?

Answer: 31

Explanation: From the information given, we can write 2 equations: $J + 63 = G$ *and* $G - 10 = 2(F - 10) = 4(J - 10)$ *. We can substitute J+63 in for G, getting* $J+53 = 2F - 20$ *and* $2F - 20 = 4J - 40$ *. Further substitution allows you to get* $J+53=4J-40$ *. Solving for J yields 31.*

6. (4) Solve for x: $x = \sqrt{506 + \sqrt{506 + \sqrt{506 + ...}}}$

Answer: 23

Explanation: If you square both sides, you get. $x^2 = 506 + \sqrt{506 + \sqrt{506 + \sqrt{506 + ...}}}$ *We know that the long line of radicals is equal to x, so this can be simplified to* $x^2 = 506 + x$ *or* $x^2 - x - 506 = 0$ *. Solving for this quadratic yields x=23. (-22 is an extraneous solution.)*

7. (5) Joe is deciding between 3 cell phone service providers: US Mobile, AT&G, and Horizon Wireless. US Mobile charges $0.10 per minute of phone calls, $0.15 per text, and $0.75 per minute of internet wifi use. AT&G charges $0.20 per minute of phone calls, $0.05 per text, and $0.50 per minute of internet wifi use. Horizon Wireless charges $0.25 per minute of phone calls, a flat rate of $20 per month for unlimited texting, and $0.20 per minute of internet wifi use. Joe averages 30 minutes of phone calls, 250 texts, and 2 hours of internet wifi use per month. Assuming Joe goes with the service provider that will give him the least cost (based on his average cell phone use), how much will his average phone bill be? (The cost can be expressed in the form a + b/100 ; leave your answer as ab. Example: Express the answer of $12.63 as 1263.)

Answer: 5150

Explanation: Letting p equal the number of minutes of phone calls per month, t equal the number of texts per month, and i equal the number of minutes of internet wifi use per month, we get an equation for each service provider: $C_{US\ Mobile} = .1p + .15t + .75i$ $C_{AT\&G} = .2p + .05t + .5i$ $C_{Horizon\ Wireless} = .25p + .2i + 20$ *. Plugging in the values for p, t, and i as 30, 250, and 120, respectively, we get the lowest cost from Horizon Wireless with a monthly bill of $51.50, expressed as 5150.*

8. (5) A store sells dorks, torques, and storks. Bob bought 3 torques and 5 dorks for $25. Kathy bought 8 storks, 21 dorks, and 3 torques for $153. Rich Joe bought 100 dorks and 2000 torques while returning (and being refunded for) 2 storks for a net total of money spent of $10176. How much would Billy pay for 1 stork, 1 torque, and 1 dork?

Answer: $19

Explanation: Set up a system of equations. These will look like:

$$5d + 3t = 25$$
$$21d + 3t + 8s = 153$$
$$100d + 2000t - 2s = 10176$$

From there, you can solve for each variable using substitution and elimination. You should find that $d = 2$, $t = 5$, and $s = 12$.

9. (6) At 12:00, Tyler is the 324th person in line to ride the "Train of the Insane" rollercoaster. Each roller coaster train holds 32 people. A full train leaves every 3 minutes. If the first 32 people in line leave on the 12:01 train, what time will Tyler's train leave? (The time can be expressed in the form a:b ; express your answer as ab. Example: Express the answer of 7:00 as 700.)

Answer: 1231

Explanation: First you can determine which group Tyler will be in -> 324/32 = 10 R4 , so he will be in the 11th group. Since he is in the 11th group, and the 1st group leaves at 12:01, he will leave 10 x 3 = 30 minutes later, at 12:31. The concatenation of this is 1231.

10. (6) Braxton enters a classroom at exactly 10:00 am and notices that the 12-hour analog clock on the wall is behaving strangely. The clock reads 12:00 and the second hand makes a complete rotation every 6 seconds. The minute hand and hour hand continue to behave as if every full rotation of the second hand indicates that a minute has passed. When Braxton leaves the class at 11:30 am, what time will the wall clock read? (The time can be expressed in the form a:b ; express your answer as ab. Example: Express the answer of 7:00 as 700.)

Answer: 300

Explanation: First you can find the number of revolutions the second hand will make within the time Braxton is in the class -> 90 x 60 = 5400 seconds -> 5400/6 = 900 revolutions = 900 "minutes". 900 minutes = 15 hours, so if the clock starts at 12:00 it will read 3:00 in 15 "hours". The concatenation of this is 300.

JACAMaC Individual Geometry Questions- 10 Questions, 45 minutes, no calculator, point values of questions are in parenthesis.

1. (3) An equilateral triangle and a regular hexagon both have the same side length. What is the ratio of the area of the triangle to the ratio of the hexagon? (For an answer $\frac{a}{b}$ express your answer as ab. Ex. Express the answer of $\frac{17}{14}$ as 1714.)

Answer: 16

Explanation: Since all interior angles in a regular hexagon measure 120^o, drawing 3 diagonals from opposite vertices through the hexagon's center will divide the hexagon into 6 triangles. Since the 120^o angles have been bisected by the diagonals to form 60^o angles, all 6 of those triangles are equilateral. Since all of those triangles share a side with the hexagon, and they are equilateral, they are all congruent to the original equilateral triangle mentioned in the problem. Therefore, since the regular hexagon can fit 6 of them, the ratio of the areas is ⅙. Concatenation of ⅙ is 16.

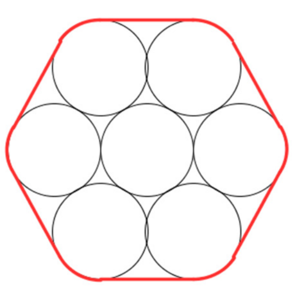
2. (3) A farmer wants to build a rectangular fence that has a length to width ratio of 2:3. He can use up to 75 feet of fencing. What is the maximum area which he can fence in? (Round your answer to the nearest whole number.)

Answer: 338

Explanation: We know that the perimeter of a rectangle is 2L + 2W. We also know that 2L = 3W. This means that the perimeter is equal to 3W + 2W = 5W = 75. Solving for W yields W=15. Plug W back into the ratio to solve for L -> 2L = 3(15) = 45. L=22.5 . The area of the rectangle is L x W = 15 x 22.5 = 337.5 , which rounds up to 338.

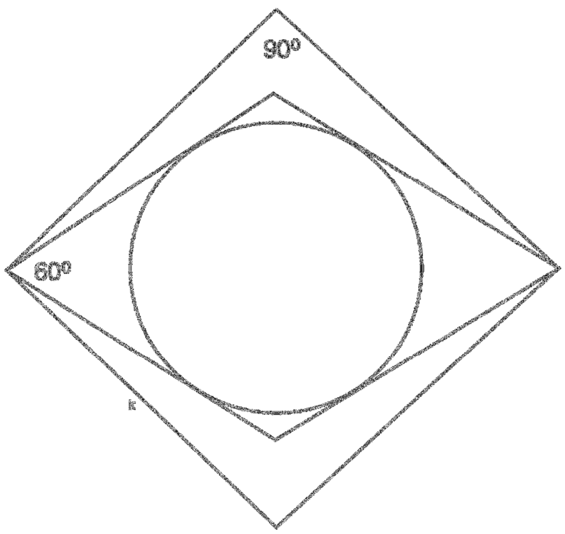
3. (3) A circle with radius 10 is surrounded by and tangent to 6 congruent circles, also with radius 10 each. Each of the 6 circles is tangential to 2 of the other 5 circles as well as the center circle. A belt wrapped tightly around the outside of the 6 circles, as shown. What is the length of the belt? (The length can be expressed as a + b ; what is a + b?)

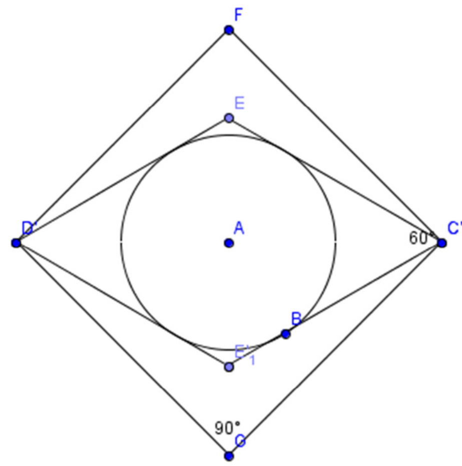
Answer: 140



Explanation: Each curved section is congruent and each straight section is congruent. Each curved section is □ of the circumference of one of the circles, and since there are 6 curved sections, the length of the curved sections total the circumference of one of the circles, or 20 . The straight sections are all tangent to 2 circles each, each stretching a length of two radii, or 20, so the total length of 6 segments is 120. The total length of the belt is then 120 + 20 , so the answer is 120 + 20 = 140

4. (3) A circle is inscribed in a rhombus with one angle measuring 60 degrees. Another rhombus with side length 16 and an angle measuring 90 degrees is circumscribed around the first rhombus so that 2 vertices of the first rhombus touch 2 vertices of the second rhombus. Find the radius of the circle. (The radius can be expressed in the form $a\sqrt{b}$;express your answer as a + b.)

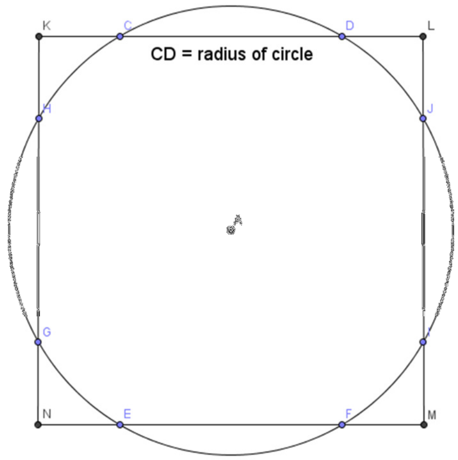




Answer: 6

Explanation: CG = GD = 16. Therefore CD = 16$\sqrt{2}$. CA is half that so CA = 8$\sqrt{2}$. Because ABC is a 30-60-90 triangle, and AC, the hypotenuse, equals 8$\sqrt{2}$, AB = 4$\sqrt{2}$. 4 + 2 = 6.

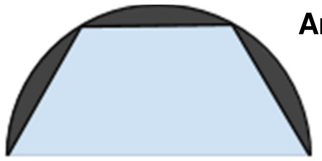
5. (4) A circle and a square intersect so that each side of the square contains a chord of the circle equal in length to the radius of the circle. What is the ratio of the area of the circle to the area of the rectangle? (The ratio can be expressed in the form $\frac{a\pi}{b}$; what is a + b ?)



Answer: 4

Explanation: Since CD = r , AC = r , and AD = r , ACD is an equilateral triangle. Due to 30-60-90 triangle properties, the altitude of that triangle is ½ r ‾ . That altitude is ½ the side length of the square. The area of the square is therefore (r ‾)^2 = 3r^2 . The area of the circle is r^2 . The ratio of their area is therefore r^2 : 3r^2 , but since the r^2 cancels out, the ratio is just / 3 . a = 1 and b = 3, so the answer is 4.

6. (4) An isosceles trapezoid is inscribed in a semicircle as shown below, such that the 3 shaded regions are congruent. The radius of the semicircle is 2. What is the area of the trapezoid? (The area can be expressed in the form $a\sqrt{b}$; what is $a \times b$?)



Answer: 9

Explanation: The 3 shaded regions are congruent, and the trapezoid is isosceles, so the legs and top base are congruent in length. Therefore, you can divide the trapezoid into 3 congruent equilateral triangles, each with side length equal to the radius (2). Now you can figure out the height of each triangle -> $h^2 + 1^2 = 2^2$ -> h = ¯. Therefore, the area of the trapezoid is ½ (4 + 2) (¯) = 3 ¯, with the answer being 3 x 3 = 9.

7. (5) A rectangular prism has a volume of 405 $units^3$. The ratio of length : width : height of the prism is 1:3:5 . What is the prism's surface area?

Answer: 9

Explanation: The 3 shaded regions are congruent, and the trapezoid is isosceles, so the legs and top base are congruent in length. Therefore, you can divide the trapezoid into 3 congruent equilateral triangles, each with side length equal to the radius (2). Now you can figure out the height of each triangle -> $h^2 + 1^2 = 2^2$ -> h = ¯. Therefore, the area of the trapezoid is ½ (4 + 2) (¯) = 3 ¯, with the answer being 3 x 3 = 9.

8. (5) A 5-unit line segment rotates 90^o about one of its endpoints every second. The location of that endpoint moves one unit to the left every second. How much area is swept out by the line segment during the course of 4 seconds? (Area swept out is defined by all area covered by the line during its course, including the moment it started and the moment it ended. The area can be expressed as the product of a and π what is a ?)

Answer: 25

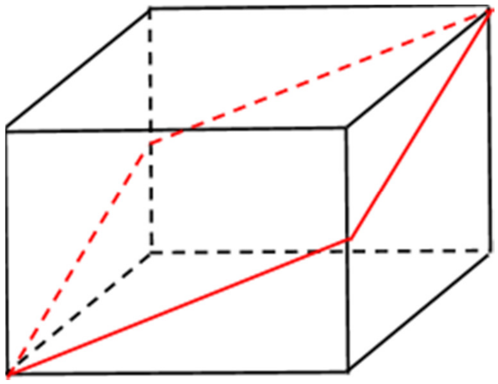
Explanation: The area swept out, if you draw it out, will be half of an ellipse with major radius 6 and minor radius 5, plus half of an ellipse with major radius 5 and minor radius 4. Using the formula for areas of ellipses, the total area is (1/2)(5)(6) + (1/2)(5)(4) = 25 $units^2$. a = 25.

9. (6) A hexagonal pyramid and a triangular prism have the same volume. The hexagonal base of the pyramid is equilateral and equiangular, and the height of the pyramid is equal to the length one of the hexagonal base's sides. The triangular faces of the prism are equilateral, and the height of the prism is equal to the length one of the triangular base's sides. What is the ratio of the side length of the pyramid's hexagonal face to the side length of the prism's triangular face? (The ratio can be expressed in simplest form as $\frac{\sqrt[a]{b}}{c}$; what is $a \times b \times c$?)

Answer: 6

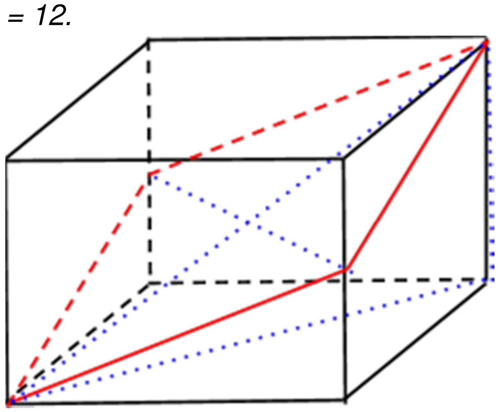
Explanation: Since their areas are the same, we can set the 2 volume formulas for the objects equal to each other -> $B_1h_1 = \frac{1}{3} B_2h_2$ *.* B_1 *(the equilateral triangle) has an area of ½ bh. Due to 30-60-90 triangle ratios, we know that if we assign* s_1 *as the side length of the triangle, we can set ½ b =1/2* s_1 *and* $h = \frac{1}{2} s_1 \sqrt{3}$*. Therefore, the area of the triangle is* $\frac{1}{4} s_1^2 \sqrt{3}$*, and since* $h_1=s_1$ *, the volume of the prism is* $\frac{1}{4} s_1^3 \sqrt{3}$*.* B_2 *(the regular hexagon) has an area of ½ asn. Due to the nature of regular hexagons, their apothems are equal to* $\frac{1}{2} s\sqrt{3}$ *(30-60-90 ratios). If we assign* s_2 *as the side length of the hexagon, its area is* $\frac{1}{2} (\frac{1}{2} s_2 \sqrt{3})(s_2)(6) = 3/2 s_2^2 \sqrt{3}$*. Since* $h_2 = s_2$ *, the volume of the pyramid is* $\frac{1}{2} s_2^3 \sqrt{3}$*. Setting the 2 volumes equal to each other, we can cancel out the* $\sqrt{3}$ *and multiply both sides of the equation by 4 to get* $s_1^3 = 2s_2^3$ *. Cube-rooting both sides gets* $s_1 = s_2 \sqrt[3]{2}$ *, or a* $1:\sqrt[3]{2}$ *ratio of side lengths (triangle's:hexagon's). Remember to write the ratio in the correct order,* $\sqrt[3]{2} : 1$*, and then multiply the values 3 x 2 x 1, to get a final answer of 6.*

10. (6) A cube is sliced by a plane which goes through 2 opposite corners and the midpoint of 2 edges as shown. If the cube has edge length 2, what is the area of the rhombus formed by the intersection of the plane and the cube? (The area can be expressed in the form $a\sqrt{b}$; what is $a \times b$?)



Answer: 12

Explanation:The area of a rhombus ½ d_1 x d_2 . The shorter diagonal (d_1) is the line between the midpoints of the opposite edges and is equal to the diagonal of one of the square faces of the cube -> $d_1^2 = 2^2 + 2^2 = 8$, d_1 = ¯ = ¯. The longer diagonal (d_2) is the line between the opposite vertices of the cube and is equal to the hypotenuse of a right triangle whose sides are an edge of the cube and a diagonal of a square face (which is equal to d_1) -> $d_2^2 = 2^2 +$ ¯ ^{2} = 12, d_2 = ¯. So, the area of the rhombus is ½ (¯) x (¯) = 2 ¯, with the answer being 2 x 6 = 12.



JACAMaC Individual General Questions - 10 Questions, 45 minutes, no calculator, point values for each question are in parentheses.

1. (3) Bob is thinking of a number. This number is equal to the sum of the first 5 positive integers that leave a remainder of 7 when divided by 13. What is Bob's number?

Answer: 230

Explanation: Add the first 5 multiples of 13 to 7 (13+7, 26+7, 39+7, 52+7, 65+7) This way, when the number is divided by 13, you will be left with a remainder of 7. This problem can be done with any remainder and integer. Now, add the values together -> 20 + 33 + 46 + 59 + 72 = 230.

2. (3) What are the last two digits of 5^{2013}?

Answer: 25

Explanation: You should see a pattern if you do the first couple of powers of 5. 5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625. The last two digits of powers of 5 greater than or equal to 2 is 25, therefore the last two digits of 5^2013 are 25.

3. (3) The last two digits of a perfect square repeat in a pattern such that if 2 numbers add to a multiple of 50, each of their squares end in the same two digits. (A perfect square such as 1, 4 and 9 are considered to end in 01, 04, and 09, respectively.) What is the largest number less than 100 whose perfect square ends in 89?

Answer: 83

Explanation: If you memorize your perfect squares, you will know that 17^2 = 289. Due to the rule mentioned above, you will know that a number that adds with 17 equals a multiple of 50, their square ends in 89. Since 100 is a multiple of 50, and 17 is the smallest number whose square ends in 89, the largest number less than 100 whose square ends in 89 is 100 - 17 = 83.

4. (3) The addition problem below has a unique solution. Each of the letters, A, E, L, M and S represents a different nonzero digit. What would the sequence MESA represent?

$$\begin{array}{r} S\,E\,A\,L \\ +S\,E\,A\,L \\ \hline L\,L\,A\,M\,A \end{array}$$

Answer: 4652

Explanation: L = 1 because the sum of two 4 digit numbers can only have a 1 in the fifth digit. Therefore A = L + L = 1 + 1 = 2 and M = A + A = 2 + 2 = 4. We then get that 112 = SE + SE = 2SE. SE = 56 so S =5 and E=6. Therefore MESA = 4652.

5. (4) The positive, even integers are written consecutively in the pattern below. What integer will be the 6th entry in Row B?

Row A				8						20		
Row B			6		10				18		22	
Row C		4				12		16				. . .
Row D	2						14					

Answer: 34

Explanation: In Row B, the pattern can be found that for every odd term (as in the 1st, 3rd, 5th, etc) that it starts at 6 and increases by 12 each time. For every even term (2nd, 4th, 6th, etc) the pattern is 12(n/2 -1) + 10. So, since we are looking for the 6th term in Row B, and 6 is even, the answer is 12(6/2 - 1) + 10 or 34.

6. (4) Numbers on a standard 6-faced die are arranged such that numbers on opposite faces always add to 7. The product of the numbers appearing on the 4 lateral faces of a rolled die is calculated (ignoring the numbers on the top and bottom). What is the minimum possible value of this product?

Answer: 60

Explanation: In order to get the minimum value, you need to find which combinations of values work. If 1 and 6 are designated as the top and bottom, the product is 5 x 4 x 3 x 2 = 120. If 2 and 5 are designated as the top and bottom, the product is 6 x 4 x 3 x 1 = 72. If 3 and 4 are designated as the top and bottom, the product is 6 x 5 x 2 x 1 = 60, which is the smallest product.

7. (5) For a certain natural number n, n^2 gives a remainder of 1 when divided by 5, and n^3 gives a remainder of 1 when divided by 5. What remainder does n give when divided by 5?

Answer: 1

Explanation: If n^2 and n^3 give a remainder of 1 when divided by 5, n^2 and n^3 must either end in 1 or 6. Numbers that, when squared, end in 1 must end in 1 or 9. However, when 9 is cubed it ends in 9, so n cannot end in 9. Numbers that, when squared, end in 6 must end in 4 or 6. However, when 4 is cubed it ends in 4, so n cannot end in 4. n must end in either 1 or 6, and if either is divided by 5, the remainder is 1.

8. (5) Kathy has 20 quarters, 50 dimes, and 100 nickels. How many different ways can she make $5 with only the coins she has?

Answer: 541

Explanation: You can write out all the possibilities, but that will take too long. While writing out the first few possibilities, you should be able to discover a pattern.

20Q						*(1 way)*
19Q 2D 1N	*19Q 1D 3N*	*19Q 5N*				*(3 ways)*
18Q 5D	*18Q 4D 2N*	*18Q 3D 4N*	*18Q 2D 6N*	*18Q 1D 8N*	*18Q 10N*	*(6)*
17Q 7-0D						*(8 ways)*

…

When there is an odd number of quarters, there are $3+5((19-x)/2)$ combinations (where $x=$ number of quarters). When there is an even number of quarters, there are $1+5((20-x)/2)$ combinations. There are 21 different numbers of quarters (11 even and 10 odd). Find the sum of each of the series -> ($3+8+13+18...+43+48 = 255$) and ($1+6+11+16...+46+51=286$). Add these together to get your final answer -> 541.

9. (6) A video game developer wants to test their new game. In a certain level in their game, a player controls a character which exists in an environment with an infinite number of rooms. Each room has 5 tunnels leading out of it, 1 of which leads to the victory room, while the other 4 lead to other rooms. A player who enters the victory room instantly gains 50 points, and the level is passed. A player who enters a tunnel which does not lead to the victory room instantly loses 5 points, and once leaving a tunnel cannot find their way back. If a player starts the level with 100 points, how many points would they be expected (or estimated) to have after passing the level?

Answer: 130

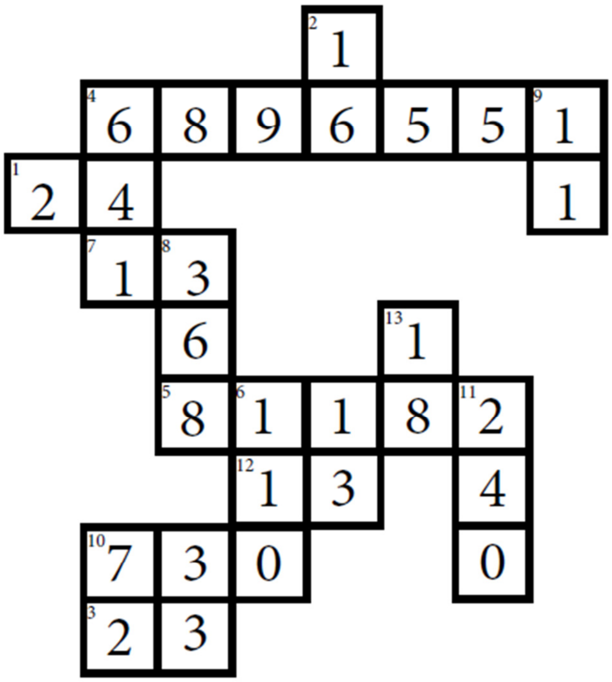
Explanation: It is expected that if there are 5 tunnels, and a 1 in 5 chance of choosing the correct tunnel, then the player will on average incorrectly choose 4 tunnels and correctly choose the 5th, giving them $100 - 4(5) + 50$ or 130 points.

10. (6) Joe uses a spinner to select a number from 1 through 5, each with equal probability. Bob uses a different spinner to select a number 6 through 10, each with equal probability. What is the probability that the product of Joe's number and Bob's number is less than 20? (The probability can be expressed as the common fraction $\frac{a}{b}$; express your answer as ab. Example: Express the answer of $\frac{14}{17}$ as 1417.)

Answer: 25

Explanation: First, you will find that there are 5 x 5 or 25 total combinations for spinner results. Now, you must find how many results will have a product less than 20. If Joe spins a 1, Bob can spin any number 6 through 10 for a total of 5 combinations. If Joe spins a 2, Bob can spin any number 6 through 9 for a total of 4 combinations. If Joe spins a 3, Bob can spin a 6 for a total of 1 combination. If Joe spins a 4 or 5, anything Bob spins will have a product more than 20. So, the probability is (5 + 4 + 1)/25 or ⅖ , with a concatenation of 25.

2013 JACAMAC Team Round



1) Across

What is the greatest positive integer n such that 5^n is a factor of 100! ?

Answer: 24

Explanation: First you need to find out how many multiples of 5 exist in 100! . 100/5 = 20 multiples. However, we need to find out how many multiples of 25 (which has 2 5's in it) exist in 100! -> 100/25 = 4. The next power of 5 is 125 which is too large, so there are 24 5's in 100!, so n = 24.

2) Down

During the years between 1962 and 2013 (inclusive), there have been 1.7 x 10^9 card games per year which have used a standard 52-card deck. Assuming that the probability plays out exactly, how many of these card games have had 4 aces (any order) in a row within a deck? (The amount can be expressed as ; what is a ?)

Answer: 16

Explanation: First consider the 4 aces as one single entity. The number of ways to arrange that in a deck is 49! ways (since there are 48 other cards plus the unity of aces). The number of ways to then arrange the 4 aces within their unity is 4! ways. The probability of 4 aces being together in a deck is therefore (49! x 4!)/(52!), or simplified out to (4!)/(52 x 51 x 50) since you can factor out 49! from 52!. You can (and probably should) write out (4!)/(52 x 51 x 50) as (4/52)(3/51)(2/50) or (1/13)(1/17)(1/25). Now you can multiply this probability times the number of card games per year and the number of years, but factor out common factors before multiplying. (1/13)(1/17)(1/25)(1.7)(10^9)(52) = (4)(.1)(4)(10^7) = 1.6 x 10^7, which equals 16 x 10^6 , meaning the answer is 16.

3) Across

A farmer named Xenu who lives on the planet Beeblebrox raises 2 types of creatures: Poggles and wingbats. Poggles are creatures with 4 legs and 2 eyes. Wingbats are creatures with 2 legs and 4 eyes. On Xenu's farm, there are a total of 66 legs and 72 eyes. How many creatures are on Xenu's farm?

Answer: 23

Explanation: You can write 2 equations from this question -> $4P + 2W = 66$ and $2P + 4W = 72$. Solving for this system of equations, we get $P=10$ and $W=13$, and the total number of creatures being 23.

4) Across

A seven digit number, with a 7 placed to the right of it, is three times smaller than the same seven digit number with a 20 placed to the left of it. What is the seven digit number?

Answer: 6896551

Explanation: The number _ _ _ _ _ _ _ 7 x 3 = 20 _ _ _ _ _ _ _ . Since 7 x 3 =21 , the last digit of the number is 1 -> _ _ _ _ _ _ 17 x 3 = 20 _ _ _ _ _ _ 1 . By multiplying and solving for the next digits, you will discover the number is 6896551.

4) Down

P(n) represents the probability that an "n" is rolled on a die. A 6-faced die, with faces 1 through 6, is weighted such that:

$P(1) = P(3)$

$P(2) = P(4) = 2(P(5))$

$P(4) = 3(P(3))$

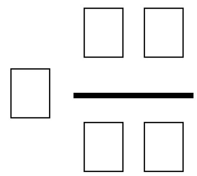
$P(5) = 2(P(6))$

If this die is rolled once, what is the probability that a "5" is rolled? (For an answer a/b, express your answer as ab. Example: Express the answer of 17/14 as 1714.)

Answer: 641

Explanation: You are looking to solve for P(5). You know that $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$. You also know that $P(6) = \frac{1}{2}(P(5))$, $P(4) = 2(P(5))$, and $P(2) = 2(P(5))$, so those can be substituted into the original equation. $P(4) = 3(P(3)) = 2(P(5))$, so $P(3) = \frac{2}{3}(P(5))$. Since $P(1) = P(3)$, P(1) also is equal to $\frac{2}{3}(P(5))$. Now you can substitute in all values into the original equation and solve for P(5) -> $\frac{2}{3}(P(5)) + 2(P(5)) + \frac{2}{3}(P(5)) + 2(P(5)) + P(5) + \frac{1}{2}(P(5)) = 1$ -> $(41/6)(P(5)) = 1$ -> $P(5) = 6/41$, concatenated to 641.

5) Across
Each of the 5 digits 2, 3, 7, 8, 9 is placed in one of the boxes to form a mixed fraction as shown below. What is the largest value of all the mixed fractions that can be formed (assuming that when the values are placed, the numerator in the fractional part is not larger than the denominator)? (The number can be expressed as the common fraction a/b ; express your answer as ab. Example: Express the answer of 17/14 as 1714.)



Answer: 81182
Explanation: First, it is obvious that the largest value will be achieved when the 9 goes in the leftmost box. Then, we are looking for the largest value of the fraction part that is not improper using the values 2, 3, 7, and 8. Therefore, we are looking for a numerator and denominator with the smallest difference yet with the largest value. The possible values with the least difference are {28, 37} and {73, 82}. Since (1 - 9/82) is larger than (1 - 9/37), we want to use 73/82 as our fraction part; our mixed fraction is therefore $9\frac{73}{82}$*. Converting this to a common fraction and concatenating it yields the answer to be 81182.*

6) Down
If $x + 1/x = 5$, then what is the value of $x^3 + 1/x^3$?
Answer: 110
Explanation: If you know that $x + 1/x = 5$*, you can see that* $(x + 1/x)^3 = 125$*. Foiling out* $(x + 1/x)^3$ *yields* $125 = x^3 + 3x + 3/x + 1/x^3$ *. You know that* $x + 1/x = 5$*, so* $3x + 3/x = 15$*, meaning that* $x^3 + 1/x^3 = 110$*.*

7) Across
Points A(-1,-2), B(5,-2), C(5,8), and D(-1,8) are vertices of rectangle ABCD, and E is on segment CD at (2,8). What is the ratio of the area of triangle ADE to the area of quadrilateral ABCE? (The ratio can be expressed as the common fraction a/b ; express your answer as ab. Example: Express the answer of 17/14 as 1714.)
Answer: 13
Explanation: Drawing out the vertices of rectangle ABCD, you can see that it has width 6 and height 10. E bisects CD, so the triangle ADE has base 3 and height 10, with an area of 15. Quadrilateral ABCE is a trapezoid with base lengths 3 and 6, and a height of 10, for an area of 45. The ratio is therefore 15/45 or $\frac{1}{3}$*, which concatenates to 13.*

8) Down

The function $a \lozenge b$ is defined as $a^b + b^a$ What is $3 \lozenge 5$?

Answer: 368

Explanation: Plug in the numbers 3 and 5 into the equation, resulting in $3^5 + 5^3$. Simplifying this out yields 243 + 125 = 368.

9) Down

Given the equations 2x + 3y = 18 , 2x + 3y + 4z = 17 , and 3x + y + 3z = 42 , what is the value of the sum x + y + z ?

Answer: 11

Explanation: First you can set up the equations like this:

2x + 3y + 0z = 18

2x + 3y + 4z = 17

3x + 1y + 3z = 42

You could solve for each variable, but if you notice the pattern, you can add all the values in each column to get 7x + 7y + 7z = 77 -> 7(x + y + z) = 77 -> x + y + z = 11.

10) Across

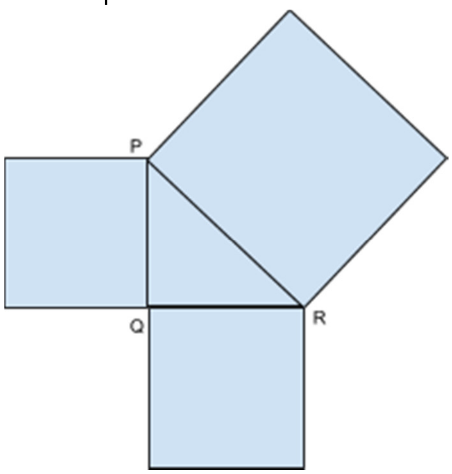
How many full 7-day weeks are there in 14 consecutive years? (Assume the 1st day of the 1st year is the 1st day of the week, and that there are no leap years.)

Answer: 730

Explanation: To find the number of weeks, multiply the number of years by the number of days in each year and then divide by 7 -> (14 x 365) / 7 -> 2(365) -> 730.

10) Down

Angle PQR is a right angle. The 3 quadrilaterals shown are squares. The sum of the areas of the 3 squares is 144. What is the area of the largest square?



Answer: 72

Explanation: You know that the sum of the 3 squares is 144, so you can represent that as $a^2 + b^2 + c^2 = 144$*. You also know that in a right triangle,* $c^2 = a^2 + b^2$*, so substituting that in the original equation you can get that* $2c^2 = 144$*, so* $c^2 = 72$*.*

11) Down

In a 4-digit number, the thousands digit is divisible by 3, the hundreds digit is greater than 4, the tens digit is greater than 2 but at most 6, and the ones digit is a prime number less than 10. How many 4-digit numbers satisfy these conditions?

Answer: 240

Explanation: The thousands digit can be represented by the set {3, 6, 9}, which has 3 possible values. The hundreds digit can be represented by the set {5, 6, 7, 8, 9} which has 5 possible values. The tens digit can be represented by the set {3, 4, 5, 6} which has 4 possible values. The ones digit can be represented by the set {2, 3, 5, 7} which has 4 possible values. Therefore, the amount of numbers that satisfy such restrictions is 3 x 5 x 4 x 4 or 240.

12) Across

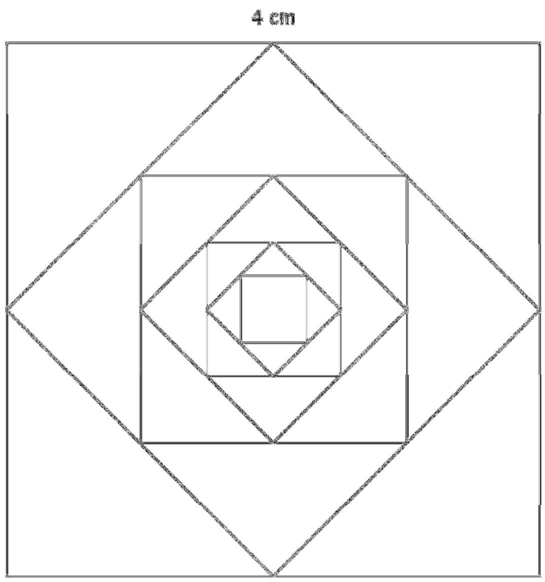
A square is to have each of its corners randomly labeled A, E, M, and S. What is the probability that, when starting at the corner labeled M, and going counterclockwise or clockwise, that it spells MESA? (The probability can be expressed as the common fraction a/b ; express your answer as ab. Example: Express the answer of 17/14 as 1714.)

Answer: 13

Explanation: There are 4 x 3 x 2 x 1 = 24 ways to label 4 corners with A, E, M, and S. Because it doesn't matter where the M is, and there are 4 corners, there are really only 24/4 = 6 ways. Since you can go either clockwise or counterclockwise, there are 2 ways to write MESA -> MESA and ASEM. The probability is therefore 2/6 = ⅓ , with a concatenation of 13.

13) Down

The side length of a square is 4 cm long. Lines are drawn between the midpoints of each side to form a second square. The process of joining the midpoints of the sides of the innermost square is repeated. What is the perimeter of the 7th square? Add your answer to the area of the 1st square.



Answer: 18

Explanation: Due to isosceles right triangle properties, you can figure out that the length of each side of the 2nd square is 2 ¯ . Then, the length of each side of the 3rd square is 2. This pattern continues -> ¯ , 1 , ½ ¯ , ½ . The perimeter of the 7th square is therefore 2. The area of the 1st square is 16, so the answer is 18.

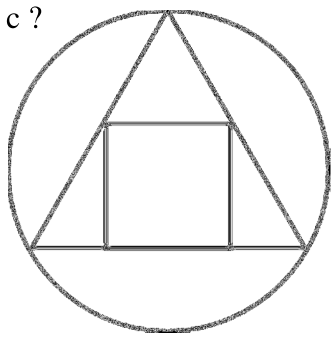
Individual Final Round Questions - 5 Questions, 30 minutes,

1) Find the number of five-digit positive integers, n, that satisfy the following conditions:
   (a) the number n is divisible by 5
   (b) the first and last digits of n are equal, and
   (c) the sum of the digits of n is divisible by 5

Answer: 200

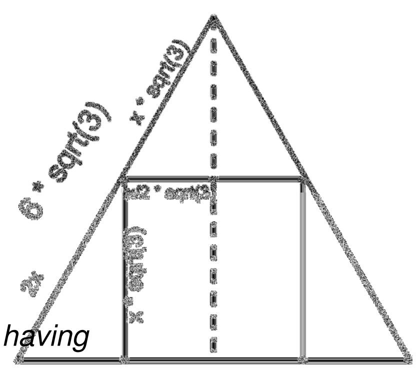
Explanation: Since the number must be divisible by 5, it must end in 0 or 5. Since the first and last digits are equal, and 0 can't be the first digit, the number must be in the form 5 _ _ _ 5. Since the sum of the digits must be divisible by 5, the 3 middle digits must add to either 0, 5 , 10, 15, 20, or 25. Trying all the possibilities, there is 1 way for the digits to sum to 0, 21 ways for the digits to sum to 5, 63 for 10, 73 for 15, 36 for 20, and 6 for 25. Adding all of those together gives a total of 200 values for n.

2) A circle is circumscribed around an equilateral triangle, and a square is then inscribed in the triangle. The area of the circle is 36 units^2. The area of the square can be written in the form $a+b\sqrt{c}$, what is a + b + c ?



Answer: 975

Explanation: Since the circle has an area 36 , its radius must be 6. On an equilateral triangle, the circumcenter is the same as the center of gravity, which the median passes through. From the ratios within a median, the larger part is equal to the radius (6), while the smaller part has length 3. Since the median is also the altitude, the side length of the triangle is 6 (this was found using 30-60-90 ratios). To find the side length of the square, you can assign variables to the sides of the triangle, as shown below:

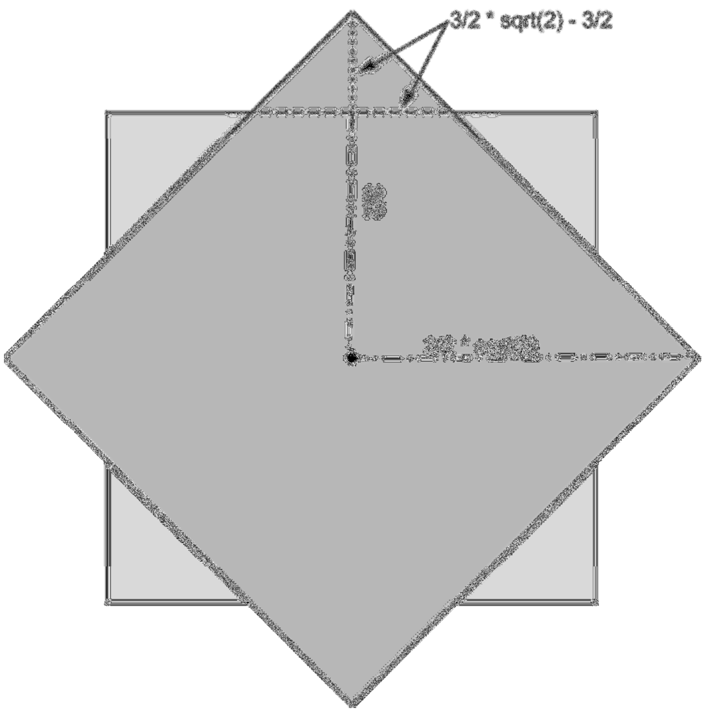


Solving for x yields x = or , with the side of the square having length and area of 2268 - 1296 . Therefore, the answer is 2268 - 1296 + 3 = 975.

3) On a standard 3x3x3 Rubik's Cube with normal rotating axes, what is the maximum surface area that can be achieved in the optimal position? The surface area can be written in the form $a + b\sqrt{c}$; what is a + b + c ?

Answer: 92

Explanation: The maximum surface area will be achieved when the most amount of the inside is exposed; this will happen when the left and right sides are rotated 45^o about the center axis. The surface area of the Rubik's Cube will be equal to the surface area of a normal 3 x 3 x 3 cube plus the surface area of the exposed surfaces. This will give an area of 54 (for the cube) with an addition of from the 16 congruent triangles exposed, as shown below, each with area . The total surface area is therefore , so the answer is 162 - 72 + 2 = 92.



4) 1) How many integers between 1 and 1000, inclusive, can be represented as the difference of two integer squares?

5) Abby and Billy each have two fair six-sided dice. Each rolls their dice and note the product of their numbers. What is the probability that one person's product is exactly one third of the other?

6) In a video game, a player earns 100 points each time they complete a level. There are 5 different score multipliers a player can purchase: x2, x4, x6, x8, and x10, which cost 1000, 2000, 3000, 4000, and 5000 points, respectively. A player who has obtained a score multiplier has all points they earn from that time forward multiplied by a certain factor (for example, a player who has obtained the score x2 has every point they earn afterward doubled.) If a player has obtained multiple score multipliers, the factors they multiply points by are compounded (for example, if a player has obtained the score x2 and the score x6, all future points they earn will be multiplied by 2*6 or 12). What is the least amount of levels a player would have to play in order to obtain all 5 score multipliers?

Answer: 26

Explanation: There is no "quick" way that I know of to do this, however it is helpful to realize that the least number of levels needed will be from the player buying the multipliers in order from lowest to largest each time they have exactly/ slightly greater than the number of points they need to purchase it. Once this is realized, you can solve for how many completed levels will need to be played. For the score x2, you need to play 1000/100 = 10 levels, with a remainder of 0 points. Then, for the score x4, you then need to play 2000/(2 x 100) = 10 levels, with a remainder of 0 points. Next, for the score x6, you then need to play 3000/(2 x 4 x 100) = 3.75 levels, round that up to 4 levels with a remainder of 200 points. For the score x8 you then need to play (4000-200)/(2 x 4 x 6 x 100) = .79666 levels, round that up to 1 level with a remainder of 1000 points. For the score x10 you then need to play (5000-1000)/(2 x 4 x 6 x 8 x 100) = .1041666 levels, round that up to 1 level with a remainder of 34400 points. Add up the total number of levels, and you get 26.